Solution (#1749) Let

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}.$$

Using the formula found in #1748 we know that

$$\begin{aligned} e^A &= \exp\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & e-1 \\ 0 & e \end{pmatrix}; \\ e^B &= \exp\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} e & e-1 \\ 0 & 1 \end{pmatrix} \\ e^{A+B} &= \exp\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} e & 2e \\ 0 & e \end{pmatrix}. \end{aligned}$$

So

$$e^A e^B = \begin{pmatrix} 1 & e-1 \\ 0 & e \end{pmatrix}\begin{pmatrix} e & e-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} e & 2e-2 \\ 0 & e \end{pmatrix} \neq e^{A+B}$$

as required.