Solution (#224) Let a and r be real numbers with $r \neq 1$. Note that the identity

$$a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1} \tag{9.1}$$

is correct for $n = 1$ as it reads RHS $= a(r - 1)/(r - 1) = a =$ LHS.

Suppose now that (9.1) is correct when $n = N$. Then

$$
\begin{aligned}
\text{LHS}\,(n = N + 1) &= a + ar + ar^2 + ar^3 + \cdots + ar^N \\
&= \left(a + ar + ar^2 + ar^3 + \cdots + ar^{N-1}\right) + ar^N \\
&= \frac{a(r^N - 1)}{r - 1} + ar^N \qquad \text{[by hypothesis]} \\
&= \frac{a\left(r^N - 1 + r^{N+1} - r^N\right)}{r - 1} \\
&= \frac{a\left(r^{N+1} - 1\right)}{r - 1} = \text{RHS}\,(n = N + 1)
\end{aligned}
$$