

Solution (#234)

(i) $P(0)$ is true; for $n \geq 0$, if $P(n)$ is true then $P(n+2)$ is true.

$P(n)$ must necessarily be true for even n .

(ii) $P(1)$ is true; for $n \geq 0$, if $P(n)$ is true then $P(2n)$ is true.

$P(n)$ must necessarily be true when n is a non-negative power of 2.

(iii) For $n \geq 0$, if $P(n)$ is true then $P(n+3)$ is true.

As it is not clear that any initial case need be true, then we cannot conclude that any $P(n)$ at all is true.

(iv) $P(0)$ and $P(1)$ are true; for $n \geq 0$, if $P(n)$ is true then $P(n+2)$ is true.

$P(n)$ must necessarily be true for all natural numbers n .

(v) $P(0)$ and $P(1)$ are true; for $n \geq 0$, if $P(n)$ and $P(n+1)$ are true then $P(n+2)$ is true.

$P(n)$ must necessarily be true for all natural numbers n .

(vi) $P(0)$ is true; for $n \geq 0$, if $P(n)$ is true then $P(n+2)$ and $P(n+3)$ are both true.

$P(n)$ must necessarily be true for $n \neq 1$.

(vii) $P(0)$ is true; for $n \geq 0$, if $P(n+1)$ is true then $P(n+2)$ is true.

Only $P(0)$ need be true.