Solution (#239) If $x_1 \leqslant 1/3$ then $1 - x_1 \geqslant 2/3$. So the result follows for $n = 1$. If the result holds for n such numbers and

$$x_1 + x_2 + \cdots + x_{n-1} + (x_n + x_{n+1}) \leqslant 1/3$$

then

$$(1 - x_1)(1 - x_2) \times \cdots \times (1 - x_{n-1})(1 - x_n - x_{n+1}) \geqslant 2/3.$$

As

$$(1 - x_n)(1 - x_{n+1}) = 1 - x_n - x_{n+1} + x_n x_{n+1} > 1 - x_n - x_{n+1}$$

it follows that

$$(1 - x_1)(1 - x_2) \times \cdots \times (1 - x_{n+1}) \geqslant 2/3.$$

The result follows by induction.