Solution (#241) The inequality

$$S_n = \sum_{r=1}^{n} \frac{1}{r^2} \leqslant 2 - \frac{1}{n}$$

is clear when $n = 1$. If it is the case that $S_N \leqslant 2 - N^{-1}$ then

$$\begin{aligned}
S_{N+1} &= S_N + \frac{1}{(N+1)^2} \\
&\leqslant 2 - \frac{1}{N} + \frac{1}{(N+1)^2} \qquad \text{[by assumption]} \\
&\leqslant 2 - \left(\frac{(N+1)^2 - 1}{N(N+1)^2} \right) \\
&\leqslant 2 - \left(\frac{N+2}{(N+1)^2} \right) \\
&\leqslant 2 - \left(\frac{N+1}{(N+1)^2} \right) \\
&\leqslant 2 - \frac{1}{(N+1)}
\end{aligned}$$

and the proof follows by induction.