Solution (#265) (i) Let T be a set of ordered pairs (n_1, n_2) of natural numbers such that:

$$(0,0) \text{ is in } T \text{ and if } (n_1,n_2) \text{ is in } T \text{ then so are } (n_1,n_2+1) \text{ and } (n_1+1,n_2).$$

As $(0,0)$ is in T, and $(k,0)$ being in T means $(k+1,0)$, then it follows by induction that each $(n,0)$ is in T.
For a given, but arbitrary, n we have $(n,0)$ is in T and (n,k) being in T means $(n,k+1)$ is in T, by induction we have any (n,m) is in T. That is $T = \mathbb{N}^2$.
(ii) Let T be a set of ordered pairs (n_1, n_2) of natural numbers such that:

$$\text{each } (0,n_2) \text{ is in } T \text{ and if } (n_1,n_2) \text{ is in } T \text{ for every } n_2 \text{ then } (n_1+1,n_2) \text{ is in } T \text{ for every } n_2.$$

If we take $P(k)$ to be the statement: '(k, n_2) is in T for all n_2' then we may rephrase the above as

$$P(0) \text{ is true and for all } k,\ P(k) \text{ is true implies } P(k+1) \text{ is true.}$$

By induction it follows that $P(n_1)$ is true for all n_1 and hence the general pair (n_1, n_2) is in $\dot{T}$. That is $T = \mathbb{N}^2$.
(iii) Let T be a set of ordered pairs (n_1, n_2) of natural numbers such that:

$$\text{each } (n_1,0) \text{ is in } T \text{ and if } (n_1,n_2) \text{ is in } T \text{ for every } n_1 \text{ then } (n_1,n_2+1) \text{ is in } T \text{ for every } n_1.$$

This is equivalent to (ii) with the roles of n_1 and n_2 swapped.