Solution (#288) There are fifteen ways to partition $\{1,2,3,4,5,6\}$ into three pairs. To see this we can see that 1 can be paired with five others. Choosing one of the remaining four elements, it can be paired with three of the remaining elements. The last two unchosen elements then form the remaining pair. So there are $5 \times 3 = 15$ such partitions.

We have that

$$\binom{6}{2,2,2} = \frac{6!}{2!2!2!} = 90.$$

This is larger by a factor of $6 = 3!$ as this trinomial coefficient remembers the order in which the pairs are selected; it counts $(\{1,2\}, \{3,4\}, \{5,6\})$ as different from $(\{3,4\}, \{1,2\}, \{5,6\})$ even though they each lead to the same partition into three pairs.