

Solution (#292) Three dice can be rolled to get a total of 16 by getting

$$6, 6, 4 \quad \text{or} \quad 6, 5, 5,$$

in some order. There are 3 orders in which either of these might occur. The probability of getting any of these particular orders is $6^{-3} = 1/216$ and hence the required probability is

$$\frac{3+3}{216} = \frac{1}{36}.$$

Three dice can be rolled to get a total of 15 by getting

$$6, 6, 3 \quad \text{or} \quad 6, 5, 4 \quad \text{or} \quad 5, 5, 5,$$

in some order. There are respectively 3, 6 and 1 orders in which these might occur. The probability of getting any of these particular orders is $6^{-3} = 1/216$ and hence the required probability is

$$\frac{3+6+1}{216} = \frac{10}{216} = \frac{5}{108}.$$

These probabilities are respectively the coefficient of x^{16} and x^{15} in the expansion of $((x + x^2 + x^3 + x^4 + x^5 + x^6)/6)^3$.