**Solution** (#296) The inequality

$$2^n \leqslant \binom{2n}{n}$$

holds for $n = 0$ with both sides being equal to 1. Suppose that the inequality holds for $n = N \geqslant 0$. Then

$$\begin{aligned}
\binom{2N+2}{N+1} &= \frac{(2N+2)!}{\{(N+1)!\}^2} \\
&= \frac{(2N+2)\,(2N+1)}{(N+1)\,(N+1)}\binom{2N}{N} \\
&= \frac{(2N+1)}{(N+1)} \times 2 \times \binom{2N}{N} \\
&\geqslant 2 \times \binom{2N}{N} \qquad [\text{as } N \geqslant 0] \\
&\geqslant 2 \times 2^N \qquad [\text{by assumption}] \\
&= 2^{N+1},
\end{aligned}$$

and the result follows by induction.

Consider now the subsets of $\{1, 2, \ldots, 2n\}$. If we split the set into $n$ pairs

$$\{1, 2\}, \quad \{3, 4\}, \quad \cdots \quad \{2n-1, 2n\}.$$

We can immediately generate $2^n$ subsets of $\{1, 2, \ldots, 2n\}$ of size $n$ by choosing precisely one element from each pair. As there are, in total, $\binom{2n}{n}$ subsets of $\{1, 2, \ldots, 2n\}$ of size $n$ it must follow that

$$2^n \leqslant \binom{2n}{n}.$$