Solution (#311) Let n be a positive integer. If we write $c = \cos\theta$ and $s = \sin\theta$ then, by De Moivre's theorem, and the binomial theorem

$$\begin{aligned}\cos n\theta &= \operatorname{Re}(c+is)^n \\ &= c^n - \binom{n}{2}c^{n-2}s^2 + \binom{n}{4}c^{n-4}s^4 - \cdots \\ &= c^n - \binom{n}{2}c^{n-2}\left(1-c^2\right) + \binom{n}{4}c^{n-4}\left(1-c^2\right)^2 - \cdots\end{aligned}$$

so that

$$\cos n\theta = a_n c^n + a_{n-2}c^{n-2} + a_{n-4}c^{n-4} + \cdots$$

for integers $a_n, a_{n-2}, \ldots$.

By Example 2.20(b) the coefficient a_n of c^n equals

$$a_n = 1 + \binom{n}{2} + \binom{n}{4} + \cdots = 2^{n-1}.$$