

Solution (#312) Begin with De Moivre's theorem and the binomial theorem,

$$\sum_{k=1}^n \binom{n}{k} \sin 2k\theta = \operatorname{Im} \left(\sum_{k=0}^n \binom{n}{k} \operatorname{cis} 2k\theta \right) = \operatorname{Im} \left(\sum_{k=0}^n \binom{n}{k} (\operatorname{cis} \theta)^{2k} \right) = \operatorname{Im} \left(\left(1 + (\operatorname{cis} \theta)^2 \right)^n \right).$$