Solution (#329) Suppose that $(n+1)\,p$ is not a natural number, so that we have a single mode. From the solution to #328 we see that the mode equals

$$k = \lfloor (n+1)\,p \rfloor,$$

where $\lfloor x \rfloor$ denote the integer part (or floor) of x. This follows because $p_{k-1} < p_k$ and $p_k > p_{k+1}$ are equivalent to

$$\lfloor (n+1)\,p \rfloor < p\,(n+1) \qquad \text{and} \qquad \lfloor (n+1)\,p \rfloor > p\,(n+1) - 1$$

both of which hold. We see that the mode exceeds the mean when

$$\lfloor np + p \rfloor > np$$

and that the mean exceeds the mode when

$$\lfloor np + p \rfloor < np.$$

If we choose $p = \frac{1}{3}$ then the former holds for $n = 2, 5, 8, \ldots$ and the latter holds for $n = 1, 4, 7, \ldots$