**Solution** (#335) Let $n \geqslant r \geqslant 0$. Then

$$\begin{aligned}
\sum_{k=r}^{n} \binom{n}{k}\binom{k}{r} &= \sum_{k=r}^{n} \frac{n!}{k!\,(n-k)!} \times \frac{k!}{r!\,(k-r)!} \\
&= \frac{n!}{r!} \sum_{k=r}^{n} \frac{1}{(n-k)!\,(k-r)!} \\
&= \frac{n!}{r!\,(n-r)!} \sum_{l=0}^{n-r} \frac{(n-r)!}{(n-r-l)!\,l!} \qquad [k = l + r] \\
&= \frac{n!}{r!\,(n-r)!}\,(1+1)^{n-r} \qquad [\text{by the binomial theorem}] \\
&= 2^{n-r}\binom{n}{r}.
\end{aligned}$$