Solution (#348) These are separately straightforward induction proofs of

$$\begin{aligned} F_1 + F_3 + F_5 + \cdots + F_{2n+1} &= F_{2n+2}. \\ F_2 + F_4 + F_6 + \cdots + F_{2n} &= F_{2n+1} - 1. \\ (F_1)^2 + (F_2)^2 + \cdots + (F_n)^2 &= F_n F_{n+1}. \end{aligned}$$

For the third it may help to note

$$F_n F_{n+1} + (F_{n+1})^2 = (F_n + F_{n+1}) F_{n+1} = F_{n+2} F_{n+1}.$$