Solution (#349) Cassini's Identity is an easy check for $n = 1$ as

$$F_2F_0 - (F_1)^2 = 2 \times 0 - 1^2 = -1 = (-1)^1 .$$

Suppose now that Cassini's Identity holds for a particular n. Then

$$\begin{aligned} F_{n+2}F_n - (F_{n+1})^2 &= (F_{n+1} + F_n) F_n - F_{n+1} (F_n + F_{n-1}) \\ &= (F_n)^2 - F_{n+1}F_{n-1} \\ &= -\left(F_{n+1}F_{n-1} - (F_n)^2\right) \\ &= -(-1)^n \qquad \text{[by hypothesis]} \\ &= (-1)^{n+1} . \end{aligned}$$

The result follows by induction.