

Solution (#356) Recall that α and β both satisfy $1 + x = x^2$ to show by induction that α and β both satisfy

$$F_{m-1} + F_m x = x^m.$$

Then recall the identity $F_k = (\alpha^k - \beta^k) / \sqrt{5}$ to show

$$\sum_{k=0}^n \binom{n}{k} (F_m)^k (F_{m-1})^{n-k} F_k = \frac{1}{\sqrt{5}} \{(\alpha^m)^n - (\beta^m)^n\} = F_{mn}$$

by the binomial theorem and the first part of this exercise.