Solution (#362) Note the identity holds for $k = 1$ and any $i, j \geqslant 1$ by Proposition 2.32.
Then prove the identity for $k = 2$ and $i, j \geqslant 1$.
Assuming the identity for $n - 1 \leqslant k \leqslant n$ and $i, j \geqslant 1$ prove it for $k = n + 1$ and any $i, j \geqslant 1$.