**Solution** (#364) If $F_n$ has $r$ decimal places this means that

$$10^{r-1} \leqslant F_n < 10^r.$$

From Proposition 2.30 we have

$$\frac{\alpha^n - \beta^n}{\sqrt{5}} \quad \text{where } \alpha = \frac{1+\sqrt{5}}{2},\ \beta = \frac{1-\sqrt{5}}{2}.$$

As $\left|\beta^n/\sqrt{5}\right| < 1$ then

$$\frac{\alpha^n}{\sqrt{5}} \leqslant 10^r.$$

Hence, taking logarithms to base 10,

$$n \leqslant \frac{r + \log_{10}\sqrt{5}}{\log_{10}\alpha}.$$

As

$$\alpha^5 = \left(\frac{1+\sqrt{5}}{2}\right)^5 = \frac{176+80\sqrt{5}}{32} > \frac{176+160}{32} = \frac{336}{10} > 10,$$

then $\log_{10}\alpha > \frac{1}{5}$. So

$$n < 5\left(r + \log_{10}\sqrt{5}\right) = 5r + \log_{10}\left(25\sqrt{5}\right) < 5r + 2.$$

As $n$ and $r$ are integers then $n \leqslant 5r + 1$.