Solution (#367) We have from Proposition 2.30 that

$$F_n = \frac{\alpha^n - \beta^n}{\sqrt{5}},$$

and $L_n = 2F_{n-1} + F_n$ from #366. Then

$$L_n = \frac{2\alpha^{n-1} - 2\beta^{n-1}}{\sqrt{5}} + \frac{\alpha^n - \beta^n}{\sqrt{5}} = \frac{\alpha^{n-1}(\alpha+2)}{\sqrt{5}} - \frac{\beta^{n-1}(\beta+2)}{\sqrt{5}}.$$

As

$$\alpha = \frac{1+\sqrt{5}}{2} = \frac{2}{\sqrt{5}-1}, \quad \text{and} \quad \beta = \frac{1-\sqrt{5}}{2} = \frac{-2}{1+\sqrt{5}},$$

then $\alpha + 2 = \sqrt{5}\alpha$ and $\beta + 2 = -\sqrt{5}\beta$. So

$$L_n = \frac{\alpha^{n-1}(\sqrt{5}\alpha)}{\sqrt{5}} - \frac{\beta^{n-1}(-\sqrt{5}\beta)}{\sqrt{5}} = \alpha^n + \beta^n.$$