Solution (#379) $\circ$ is clearly commutative: given Zeckendorf representations $m=\sum_i F_{c_i}$ and $n=\sum_j F_{d_j}$ then

$$\begin{aligned} m\circ n &= \left(\sum_i F_{c_i}\right)\circ\left(\sum_j F_{d_j}\right) \\ &= \sum_{i,j} F_{c_i+d_j} \\ &= \sum_{i,j} F_{d_j+c_i} \\ &= \left(\sum_j F_{d_j}\right)\circ\left(\sum_i F_{c_i}\right) \\ &= n\circ m. \end{aligned}$$

However $\circ$ can be easily seen to be non-distributive as

$$(1+1)\circ 1 = 2*1 = F_3\circ F_2 = F_5 = 5$$

which, being odd, clearly cannot equal $1\circ 1+1\circ 1$.