Solution (#383) Note

$$\frac{1}{F_n} - \frac{1}{F_{n+2}} = \frac{F_{n+2} - F_n}{F_n F_{n+2}} = \frac{F_{n+1}}{F_n F_{n+2}}.$$

Hence for $N \geqslant 1$,

$$\sum_{n=1}^{N} \frac{1}{F_n F_{n+2}} = \sum_{n=1}^{N} \left(\frac{1}{F_n F_{n+1}} - \frac{1}{F_{n+1} F_{n+2}} \right) = \frac{1}{F_1 F_2} - \frac{1}{F_{N+1} F_{N+2}},$$

as the remaining middle terms of the sum all cancel out. Finally, letting N tend to infinity, we have

$$\sum_{n=1}^{\infty} \frac{1}{F_n F_{n+2}} = \frac{1}{F_1 F_2} = \frac{1}{1 \times 1} = 1.$$