Solution (#413) Let F denote the expected wait for heads-then-tails on two successive tosses, and G denote the expected wait for heads-then-tails given that the first throw is heads.

The first toss can be either H or T. If the first toss is H then the expected wait is G (including the first toss). If the first toss is T the the wait begins afresh so that in all the expected wait is $F+1$. As there is a half chance of each then

$$F = \frac{1}{2}(F+1) + \frac{1}{2}G,$$

which implies that $F = 1 + G$.

Assume now that the first toss is H and consider the possibilities for the second toss. If the second toss is T then the wait is over after 2 tosses. If the second is H then we are in the same situation of waiting for HT having just gotten an H – only we are now one turn later. Hence

$$G = \frac{1}{2} \times 2 + \frac{1}{2}(G+1).$$

Rearranging this gives $G = 3$ and substituting this into our first equation gives $F = 4$.

Remark: Note that the expected wait for HT is less than the wait for HH. This is not too surprising: a T derails their wait equally but HH is not as bad for the first wait as HT is for the second, which has the effect of resetting the wait completely.