Solution (#416) Let

- E_1 denote the expected number of times the coin is tossed to first get HTH, that is the expected wait for A.
- E_2 denote the expected number of times the coin is tossed to first get HTH, given that the very first toss is H.
- E_3 denote the expected number of times the coin is tossed to first get HTH, given that first two tosses are HT.

The expected wait is E_1. There is a half chance of H on the first toss and given that the expected wait is E_2. There is a half chance of T on the first toss and given that the expected wait is $E_1 + 1$ as the count has effectively been set. So

$$E_1 = \frac{1}{2}E_2 + \frac{1}{2}(E_1 + 1) \implies E_1 = E_2 + 1.$$

Given the first toss is H the expected wait is E_2. There is a half chance of H on the second toss and given that we expect the wait to be $E_2 + 1$ (as we still have an H in our favour but are now delayed one toss.) There is a half chance of T on the second toss and given that we expect the wait to be E_3. So

$$E_2 = \frac{1}{2}(E_2 + 1) + \frac{1}{2}E_3 \implies E_1 = E_3 + 2.$$

Given the first two tosses are HT the expected wait is E_3. There is a half chance of H on the third toss and given that the wait is 3 tosses. There is a half chance of T on the third toss and given that the expected wait is $E_1 + 3$ as we are back to square one, three tosses later. So

$$E_3 = \frac{1}{2} \times 3 + \frac{1}{2}(E_1 + 3) \implies E_1 - 2 = \frac{3}{2} + \frac{E_1}{2} + \frac{3}{2},$$

and solving this gives $E_1 = 10$. Similarly now let:

- e_1 denote the expected number of times the coin is tossed to first get HTT, that is the expected wait for Person B.
- e_2 denote the expected number of times the coin is tossed to first get HTT, given that the very first toss is H.
- e_3 denote the expected number of times the coin is tossed to first get HTT, given that first two tosses are HT.

Arguing as before we have

$$\begin{aligned} e_1 &= \frac{1}{2}e_2 + \frac{1}{2}(e_1 + 1) \implies e_1 = e_2 + 1. \\ e_2 &= \frac{1}{2}(e_2 + 1) + \frac{1}{2}e_3 \implies e_1 = e_3 + 2. \\ e_3 &= \frac{1}{2} \times 3 + \frac{1}{2}(e_2 + 2) \implies 2e_1 - 4 = 3 + e_1 + 1 \end{aligned}$$

and solving this gives $e_1 = 8$,

Note that the crucial difference is in the third equation where failure to get the third toss means we revert to having one toss in our favour, and not completely having to go back to square one.