

Solution (#420) Let A_n denote the number of ways to cover a $2 \times n$ grid using 2×1 rectangular tiles. If we start tiling the $2 \times n$ grid from the left, there are two ways to proceed as shown below:

1	or	1	1	...
1		2	2	...

If we begin with a vertical tile then there are A_{n-1} ways to continue; if we begin with two horizontal tiles then there are A_{n-2} ways to continue. Hence

$$A_n = A_{n-1} + A_{n-2}.$$

Clearly $A_1 = 1 = F_2$ and $A_2 = 2 = F_3$ so that $A_n = F_{n+1}$ for $n \geq 1$.