

Solution (#427) Denote the cuboid's dimensions as x, y, z with the area of the missing face being xy . Its volume is $V = xyz$ and its surface area is

$$xy + 2yz + 2xz = A.$$

If we apply the AM-GM to xy , $2yz$ and $2xz$ we find

$$\frac{A}{3} = \frac{xy + 2yz + 2xz}{3} \geq \sqrt[3]{xy \times 2yz \times 2xz} = \sqrt[3]{4V^2}.$$

Hence

$$A \geq 3(2V)^{2/3},$$

as required.