Solution (#431) Assume without any loss of generality that $0 < x \leqslant y \leqslant z$. Then $0 \leqslant x^2 \leqslant y^2 \leqslant z^2$. So by Chebyshev's Inequality

$$\left(\frac{x+y+z}{3}\right)\left(\frac{x^2+y^2+z^2}{3}\right) \leqslant \frac{xx^2+yy^2+zz^2}{3}.$$

This rearranges to

$$\frac{x+y+z}{3} \leqslant \frac{x^3+y^3+z^3}{x^2+y^2+z^2}.$$