

Solution (#444) (i) Let $\alpha = 2/7$. Then

$$\begin{aligned} [\alpha] &= 0; & \alpha_1 &= \left(\frac{2}{7} - 0\right)^{-1} = \frac{7}{2}. \\ [\alpha_1] &= 3; & \alpha_2 &= \left(\frac{7}{2} - 3\right)^{-1} = 2. \end{aligned}$$

Hence $2/7 = [0, 3, 2]$.

(ii) Let $\alpha = 13/17$. Then

$$\begin{aligned} [\alpha] &= 0; & \alpha_1 &= \left(\frac{13}{17} - 0\right)^{-1} = \frac{17}{13}. \\ [\alpha_1] &= 1; & \alpha_2 &= \left(\frac{17}{13} - 1\right)^{-1} = \frac{13}{4}. \\ [\alpha_2] &= 3; & \alpha_3 &= \left(\frac{13}{4} - 3\right)^{-1} = 4. \end{aligned}$$

Hence $13/17 = [0, 1, 3, 4]$

(iii) Let $\alpha = 217/99$. Then

$$\begin{aligned} [\alpha] &= 2; & \alpha_1 &= \left(\frac{217}{99} - 2\right)^{-1} = \frac{99}{19}. \\ [\alpha_1] &= 5; & \alpha_2 &= \left(\frac{99}{19} - 5\right)^{-1} = \frac{19}{4}. \\ [\alpha_2] &= 4; & \alpha_3 &= \left(\frac{19}{4} - 4\right)^{-1} = \frac{4}{3}. \\ [\alpha_3] &= 1; & \alpha_4 &= \left(\frac{4}{3} - 1\right)^{-1} = 3. \end{aligned}$$

Hence $217/99 = [2, 5, 4, 1, 3]$