

Solution (#445) Let $\alpha = \sqrt{5}$. Then we have $\lfloor \alpha \rfloor = 2$ and so

$$\alpha_1 = \frac{1}{\sqrt{5} - 2} = \sqrt{5} + 2.$$

Suppose for the sake of induction that $\alpha_k = \sqrt{5} + 2$ where $k \geq 1$, then

$$\alpha_k = \lfloor \alpha_k \rfloor + \frac{1}{\alpha_{k+1}} \implies \sqrt{5} + 2 = 4 + \alpha_{k+1} \implies \alpha_{k+1} = \frac{1}{\sqrt{5} - 2} = \sqrt{5} + 2.$$