Solution (#450) Let a, b be positive integers and say

$$\alpha = [a, b, a, b, a, b, \ldots] = [a, b, \alpha].$$

Then

$$\alpha = a + \frac{1}{b + \frac{1}{\alpha}}.$$

This equation rearranges to the quadratic

$$b\alpha^2 - ab\alpha - a = 0.$$

Applying the quadratic formula we see

$$\alpha = \frac{ab \pm \sqrt{a^2b^2 + 4ab}}{2b}.$$

As x is positive and $\sqrt{a^2b^2 + 4ab} > ab$ it follows that

$$\alpha = \frac{ab + \sqrt{a^2b^2 + 4ab}}{2b}.$$

Remark: It is more generally the case that an irrational α has an eventually periodic continued fraction if and only if α satisfies a quadratic equation with rational coefficients.