Solution (#480) (i) by considering the various ways P can reach a given point at time t we see that

- X_2 is distributed with probabilities on $-2,-1,0,1,2$.
$$r^2, \quad 2qr, \quad q^2+2pr, \quad 2pq, \quad p^2.$$
- X_3 is distributed on $-3,-2,-1,0,1,2,3$ with probabilities
$$r^3, \quad 3qr^2, \quad 3q^2r+3pr^2, \quad q^3+6pqr, \quad 3pr^2+3pq^2, \quad 3qp^2, \quad p^3.$$
- X_4 is distributed on $-4,-3,-2,-1,0,1,2,3,4$ with probabilities
$$r^4, \quad 4qr^3, \quad 6q^2r^2+4pr^3, \quad 12pqr^2+4q^3r, \quad q^4+12pq^2r+6p^2r^2, \quad 12p^2qr+4pq^3, \quad 6p^2q^2+4p^3r, \quad 4p^3q, \quad p^4.$$

(ii) Arguing as in #479, the probability of P being at k at time t equals the coefficient of x^k in
$$\left(\frac{r}{x}+q+px\right)^t.$$

(iii) If we write
$$\left(\frac{r}{x}+q+px\right)^t = \sum_{k=-t}^{t} p_t^k x^k,$$
then differentiating gives
$$t\left(p-\frac{r}{x^2}\right)\left(\frac{r}{x}+q+px\right)^{t-1} = \sum_{k=-t}^{t} kp_t^k x^{k-1},$$
Setting $x=1$ we see that the mean of X_t equals
$$t(p-r),$$
recalling that $p+q+r=1$. To find the mean of $(X_t)^2$, as in #479, we differentiate
$$t\left(px-\frac{r}{x}\right)\left(\frac{r}{x}+q+px\right)^{t-1} = \sum_{k=-t}^{t} kp_t^k x^k,$$
to get
$$t(t-1)\left(px-\frac{r}{x}\right)\left(-\frac{r}{x^2}+p\right)\left(\frac{r}{x}+q+px\right)^{t-2} + t\left(p+\frac{r}{x^2}\right)\left(\frac{r}{x}+q+px\right)^{t-1} = \sum_{k=-t}^{t} k^2p_t^k x^{k-1}.$$
Setting $x=1$ we get that
$$\sum_{k=-t}^{t} k^2p_t^k = t(t-1)\left(p-r\right)\left(-r+p\right)+t\left(p+r\right) = t(t-1)(p-r)^2+t(p+r).$$