Solution (#481) Let P_a denote the probability that the particle is eventually absorbed at 0 if it currently is at a. Note that

$$P_a = pP_{a+1} + qP_{a-1} \qquad \text{when } a \geqslant 1 \text{ and with } P_0 = 1.$$

The auxiliary equation

$$m = pm^2 + q$$

has roots $m = 1$ and $m = q/p$. If $p = q$ then we have the solution

$$P_a = A + Ba.$$

As $P_0 = 1$ then $A = 1$ and as the above denotes a probability for all a we must also have $B = 0$. Hence $P_a = 1$ for all a.

If $p \neq q$ we have

$$P_a = A + B\left(\frac{q}{p}\right)^a.$$

As $P_0 = 1$ then $A + B = 1$.

If $q > p$ and $B \neq 0$ then the above formula cannot denote a probability as a becomes large. Hence $B = 0$ in this case, $A = 1$ and $P_a = 1$ for all a. Absorption is certain and this seems reasonable enough as there is a clear drift to the left when $q > p$.

Say $p > q$ now. We have

$$P_a = 1 - B + B\left(\frac{q}{p}\right)^a.$$

We are told that $P_a \to 0$ as $a \to \infty$ when $p > q$ and hence we have $B = 1$. So

$$P_a = \left(\frac{q}{p}\right)^a.$$