Solution (#486) If the increasing sequence $a_1, a_2, a_3, \ldots$ is complete then, by #485,

$$a_1 = 1 \qquad \text{and} \qquad 1 + a_1 + a_2 + \cdots + a_k \geqslant a_{k+1} \quad \text{for } k \geqslant 1.$$

Note $a_1 = 1 = 2^{1-1}$. Suppose as an inductive hypothesis that $a_k \leqslant 2^{k-1}$ for $1 \leqslant k \leqslant n$. Then

$$a_{n+1} \leqslant 1 + a_1 + a_2 + \cdots + a_n \leqslant 1 + 1 + 2 + \cdots + 2^{n-1} = 2^n.$$

The result then follows by induction.