Solution (#1060) Let A be the associated matrix of a shear of $\mathbb{R}_3$. Take vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ so that $\mathbf{v}_3$ is normal to the invariant plane and $\mathbf{v}_1, \mathbf{v}_2$ are parallel to the plane and let X_1, X_2, X_3 be the corresponding co-ordinates.

With respect to these co-ordinates the invariant plane has equation $X_3 = 0$ and so, in a similar fashion to #1096, we see that the shear has associated matrix

$$\begin{pmatrix} 1 & 0 & d_1 \\ 0 & 1 & d_2 \\ 0 & 0 & 1 \end{pmatrix}$$

for some d_1, d_2. In particular we see $\text{trace}A = 3$ and $\det A = 1$, noting that neither of these changes with the change of co-ordinates.

Note from above that $c_A(x) = (x-1)^3$ for any shear A. However knowing $\text{trace}A = 3$ and $\det A = 1$ of a matrix A only specifies the x^2 and constant coefficients in $c_A(x)$. For example any matrix A with characteristic polynomial

$$c_A(x) = x^3 - 3x^2 + kx - 1$$

would have $\text{trace}A = 3$ and $\det A = 1$ but could not be a shear if $k \neq 3$.

If we take $k = -5$ so that $x = -1$ is a root we obtain

$$\begin{aligned} x^3 - 3x^2 - 5x - 1 &= (x+1)(x^2 - 4x - 1) \\ &= (x+1)\left((x-2)^2 - 5\right). \end{aligned}$$

So a matrix A which is not a shear but has $\text{trace}A = 3$ and $\det A = 1$ is

$$\text{diag}(-1, 2 - \sqrt{5}, 2 + \sqrt{5})$$

and we see that the converse does not hold.