Solution (#1080) Let

$$A = \frac{1}{35}\begin{pmatrix} 11 & -32 \\ 18 & 59 \end{pmatrix}.$$

Then

$$\begin{aligned} c_A(x) &= \frac{1}{35^2}\begin{vmatrix} 35x-11 & 32 \\ -18 & 35x-59 \end{vmatrix} \\ &= \frac{1}{35^2}\left(35^2x - (11+59)35x + (11\times 59 + 18\times 32)\right) \\ &= \frac{1}{35^2}\left(35^2x - 70\times 35x + (11\times 59 + 18\times 32)\right) \\ &= \frac{1}{35^2}(35^2x - 70\times 35x + 35^2) \qquad [\text{as } 35^2 - 24^2 = 11\times 59] \\ &= (x-1)^2. \end{aligned}$$

Hence 1 is the only eigenvalue of A and

$$(A-I)\,\mathbf{x} = \mathbf{0} \quad \iff \quad \begin{pmatrix} -24 & -32 \\ 18 & 24 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{0} \quad \iff \quad 3x+4y=0$$

and so the 1-eigenvectors are multiples of $(4,-3)^T$.

Now let

$$P = \frac{1}{5}\begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}.$$

Then $P^{-1}AP = P^TAP$ equals

$$\begin{aligned} & \left[\frac{1}{5}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}\right]\left[\frac{1}{35}\begin{pmatrix} 11 & -32 \\ 18 & 59 \end{pmatrix}\right]\left[\frac{1}{5}\begin{pmatrix} 4 & 3 \\ -3 & 4 \end{pmatrix}\right] \\ =\; & \frac{1}{5^2\times 35}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 44+96 & 33-128 \\ 72-177 & 54+236 \end{pmatrix} \\ =\; & \frac{1}{5^2\times 35}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 140 & -95 \\ -105 & 290 \end{pmatrix} \\ =\; & \frac{1}{5\times 35}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 28 & -19 \\ -21 & 58 \end{pmatrix} \\ =\; & \frac{1}{5\times 35}\begin{pmatrix} 112+63 & -76-174 \\ 84-84 & -57+232 \end{pmatrix} \\ =\; & \frac{1}{5\times 35}\begin{pmatrix} 175 & -250 \\ 0 & 175 \end{pmatrix} \\ =\; & \begin{pmatrix} 1 & -10/7 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$