Solution (#1113) Let A be an $n \times n$ matrix and $T = \mu_A$ and $S = \mu_{A^T}$. Let $\mathcal{V}$ be an orthonormal basis for $\mathbb{R}_n$ and let $\mathcal{E}$ denote the standard basis for $\mathbb{R}_n$. So we have

$$_{\mathcal{E}}T_{\mathcal{E}} = A \qquad \text{and} \qquad _{\mathcal{E}}S_{\mathcal{E}} = A^T.$$

As $\mathcal{V}$ is orthonormal then $P = {}_{\mathcal{E}}I_{\mathcal{V}}$ is orthogonal – the columns of P are the vectors in $\mathcal{V}$. We note that

$$_{\mathcal{V}}T_{\mathcal{V}} = ({}_{\mathcal{V}}I_{\mathcal{E}})\,({}_{\mathcal{E}}T_{\mathcal{E}})\,({}_{\mathcal{E}}I_{\mathcal{V}}) = P^T AP$$

and that

$$_{\mathcal{V}}S_{\mathcal{V}} = ({}_{\mathcal{V}}I_{\mathcal{E}})\,({}_{\mathcal{E}}S_{\mathcal{E}})\,({}_{\mathcal{E}}I_{\mathcal{V}}) = P^T A^T P.$$

Hence by the transpose product rule we have

$$\begin{aligned} ({}_{\mathcal{V}}T_{\mathcal{V}})^T &= (P^T AP)^T \\ &= P^T A^T P^{TT} \\ &= P^T A^T P = {}_{\mathcal{V}}S_{\mathcal{V}} \end{aligned}$$

as required.