Solution (#1152) Consider the conic

$$3x^2 + 7xy + 2y^2 + x - 3y - 2 = 0.$$

If we differentiate implicitly then we find that

$$6x + 7y + 7x\frac{\mathrm{d}y}{\mathrm{d}x} + 4y\frac{\mathrm{d}y}{\mathrm{d}x} + 1 - 3\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

and so

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x + 7y + 1}{3 - 7x - 4y}.$$

If we substitute $x = 1$ and $y = -1$ we see that the formula yields $0/0$ and so gives no sensible answer.

To understand this note that the conic's equation can be factorized as

$$(x + 2y + 1)(3x + y - 2) = 0,$$

and so the conic describes two intersecting lines, their intersection being at $(1, -1)$. So it is not unreasonable for the formula to not work at $(1, -1)$ as there are two possible gradients at that point.

Elsewhere on the conic we have one of the factors equally zero. If $x + 2y + 1 = 0$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x + 7y + 1}{3 - 7x - 4y} = \frac{-12y - 6 + 7y + 1}{3 + 14y + 7 - 4y} = \frac{-5y - 5}{10y + 10} = -\frac{1}{2}$$

which is indeed the gradient of the line $x + 2y + 1 = 0$. Likewise if $3x + y - 2 = 0$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6x + 7y + 1}{3 - 7x - 4y} = \frac{6x + 14 - 21x + 1}{3 - 7x - 8 + 12x} = \frac{-15x + 15}{5x - 5} = -3$$

which is the gradient of the line $3x + y - 2 = 0$.