**Solution** (#1161) From Theorem 4.53 and #1157 we know that

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

describes a hyperbola when $B^2 > 4AC$ and

$$k = \left(\frac{CD^2 + AE^2 - BDE}{4AC - B^2}\right) - F \neq 0.$$

Expanding $(ax + by + c)(dx + ey + f) = 1$ we see

$$A = ad, \quad B = ae + bd, \quad C = be, \quad D = af + cd, \quad E = bf + ce, \quad F = cf - 1.$$

So

$$B^2 - 4AC = (ae + bd)^2 - 4abde = (ae - bd)^2.$$

Hence we require that $ae \neq bd$. This is not surprising as this is the condition that the lines $ax + by + c = 0$ and $dx + ey + f = 0$ not be parallel.

We also have that $CD^2 + AE^2 - BDE$ equals

$$\begin{aligned} & be(af + cd)^2 + ad(bf + ce)^2 - (ae + bd)(af + cd)(bf + ce) \\ = & -b^2cd^2f + 2abcdef - a^2ce^2f \\ = & -cf(ae - bd)^2. \end{aligned}$$

Hence

$$k = \left(\frac{-cf(ae - bd)^2}{-(ae - bd)^2}\right) - (cf - 1) = 1 \neq 0.$$

So provided $ae \neq bd$ the equation otherwise describes a hyperbola.