Solution (#1168) Consider the conic C with polar equation

$$r=\frac{ke}{1+e\cos\theta}.$$

A straight line L through the focus F (which is the origin) is represented as two half lines $\theta=\alpha$ and $\theta=\pi-\alpha$, so that A and B have polar co-ordinates

$$A:r=\frac{ke}{1+e\cos\alpha},\quad \theta=\alpha,\qquad A:r=\frac{ke}{1+e\cos(\pi-\alpha)}=\frac{ke}{1-e\cos\alpha},\quad \theta=-\alpha.$$

Hence

$$\frac{1}{|AF|}+\frac{1}{|BF|}=\left(\frac{1+e\cos\alpha}{ke}\right)+\left(\frac{1-e\cos\alpha}{ke}\right)=\frac{2}{ke},$$

is independent of α and so of the choice of line L.

For the parabola $y^2=4ax$ the focus is $F=(a,0)$ and the vertical line $x=a$ meets the parabola at $(a,\pm 2a)$. Hence

$$\frac{1}{|AF|}+\frac{1}{|BF|}=\frac{1}{2a}+\frac{1}{2a}=\frac{1}{a}.$$