Solution (#1196) The associated symmetric matrix has characteristic polynomial

$$c(x) = x^3 - 6x^2 - 5x + 1.$$

We can show $c(x)$ has three real roots, one $-\gamma$ being negative and two α, β being positive.

There is an orthogonal change of variable $(x, y, z) \longmapsto (X, Y, Z)$ such that

$$f(x, y, z) = \alpha X^2 + \beta Y^2 - \gamma Z^2.$$

So $f(x, y, z) = 1$ or equivalently

$$\alpha X^2 + \beta Y^2 = \gamma Z^2 + 1$$

is a hyperboloid of one sheet, and $f(x, y, z) = -1$ or equivalently

$$\alpha X^2 + \beta Y^2 = \gamma Z^2 - 1$$

is a hyperboloid of two sheets.