Solution (#1220) We have

$$\cos a = \sin b \cos \alpha \sin c + \cos b \cos c.$$

If we are able to employ the approximations $\cos x \approx 1 - x^2/2$, $\sin x \approx x$ for $x = a, b, c$ then we have

$$1 - \frac{a^2}{2} = bc \cos \alpha + \left(1 - \frac{b^2}{2}\right)\left(1 - \frac{c^2}{2}\right).$$

Ignoring the fourth order term b^2c^2 we then have

$$1 - \frac{a^2}{2} = bc \cos \alpha + 1 - \frac{b^2}{2} - \frac{c^2}{2}$$

which rearranges to the usual

$$a^2 = b^2 + c^2 - 2bc \cos \alpha.$$

Likewise the spherical sine rule states

$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}.$$

If the triangle is small enough that we can employ the approximations $\sin x \approx x$ for $x = a, b, c$ then we obtain the usual sine rule

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}.$$