Solution (#1224) Let Δ_1 and Δ_2 be two spherical triangles with sides a, b, c. Label their vertices as A, B, C and A', B', C' so that a equals the length of BC and $B'C'$ etc.. Note that from the first cosine rule it must follow that the angles α, β, γ of the two triangles are also equal.

The points A and A' lie on some great circle and by a rotation about the normal to that plane we can move A' to A. That rotation would be effected by some orthogonal matrix. Then by rotating about an axis through $A = A'$ we can rotate the great circles containing AB and $A'B' = AB'$ so that they coincide. Further as c is the common length of the arcs AB and $AB' = A'B'$ then we now have that $B = B'$. Again some orthogonal matrix effects this.

The angles CAB and $C'AB$ both equal α. This means that either the great circle AC is the same as the great circle AC' or that ABC is a reflected version of ABC' in the plane containing the arc AB. If the latter then we can use an orthogonal matrix to effect a reflection in the plane containing AB.

We are now in a situation where the great circle AC is the same as the great circle AC' and that the arcs AC and AC' have the same length and so $C = C'$.

The overall effect of the three orthogonal matrices we have used is still that of a single orthogonal matrix (#1089) and the result follows.

Now given two similar spherical triangles, it follows immediately from the second cosine rule that their sides also have the same length. By the previous argument we then have that the triangles are congruent – that is there is an isometry of the sphere taking one to the other.