Solution (#1230) Let $X = \langle (0,2,1,2), (1,2,2,3) \rangle$. A vector (x_1, x_2, x_3, x_4) is in $X^\perp$ if it solves the system

$$\left(\begin{array}{cccc|c} 0 & 2 & 1 & 2 & 0 \\ 1 & 2 & 2 & 3 & 0 \end{array} \right).$$

Row-reducing the system, we arrive at

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & 1 & 0 \end{array} \right),$$

which has general solution

$$(x_1, x_2, x_3, x_4) = \left(-\alpha - \beta, -\frac{1}{2}\alpha - \beta, \alpha, \beta \right) = \frac{1}{2}\alpha\,(-2,-1,2,0) + \beta\,(-1,-1,0,1).$$

So $(-2,-1,2,0)$ and $(-1,-1,0,1)$ form a basis for $X^\perp$.