Solution (#1250) (i) For each t, let $A(t)$ denote an orthogonal matrix which is differentiable with respect to t. Then

$$A(t)A(t)^T = I.$$

If we differentiate with respect to t then

$$A'(t)A(t)^T + A(t)A'(t)^T = 0$$

and so

$$M(t) + M(t)^T = 0$$

where

$$M(t) = A'(t)A(t)^T.$$

So we can write

$$M(t) = \begin{pmatrix} 0 & \gamma & -\beta \\ -\gamma & 0 & \alpha \\ \beta & -\alpha & 0 \end{pmatrix}$$

for some α, β, γ which are dependent on t.

(ii) For a fixed vector $\mathbf{v}_0$ define $\mathbf{v}(t) = A(t)\mathbf{v}_0$. As $A(t)$ is orthogonal we have

$$\mathbf{v}'(t) = A'(t)\mathbf{v}_0 = A'(t)A(t)^T A(t)\mathbf{v}_0 = M(t)\mathbf{v}(t).$$

Now by #889

$$\begin{pmatrix} 0 & \gamma & -\beta \\ -\gamma & 0 & \alpha \\ \beta & -\alpha & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \wedge \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

and so we set

$$\boldsymbol{\omega}(t) = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}.$$