Solution (#1275) Let f be a complex-valued integrable function on $[a,b]$. Then $\overline{f}$ is likewise integrable. By #1272 $|f|^2 = f\bar{f}$ is integrable and then by #1273 $|f| = \sqrt{f\bar{f}}$ is also integrable.

Now let f be a complex integrable function on $[a,b]$. Let λ be a complex number of modulus 1 such that

$$\lambda \int_a^b f(x)\,\mathrm{d}x$$

is real. Then, ,

$$\begin{aligned}
\left|\int_a^b f(x)\,\mathrm{d}x\right| &= \left|\int_a^b \lambda f(x)\,\mathrm{d}x\right| \\
&= \left|\int_a^b \mathrm{Re}(\lambda f(x))\,\mathrm{d}x\right| \\
&\leqslant \int_a^b \left|\mathrm{Re}(\lambda f(x))\right|\,\mathrm{d}x \qquad \text{by (5.16)} \\
&\leqslant \int_a^b \left|\lambda f(x)\right|\,\mathrm{d}x \\
&= \int_a^b \left|f(x)\right|\,\mathrm{d}x
\end{aligned}$$

as required.