Solution (#1277) Let f, g be integrable functions on $[a, b]$. Say $m \leqslant f(x) \leqslant M$ and $g(x) > 0$ for all x. By #1272 we know that fg is integrable and as $g(x) > 0$ for each x we have

$$mg(x) \leqslant f(x)g(x) \leqslant Mg(x)$$

for all x. Hence by Proposition 5.12(b) we have

$$m \int_a^b g(x)\,\mathrm{d}x \leqslant \int_a^b f(x)g(x)\,\mathrm{d}x \leqslant M \int_a^b g(x)\,\mathrm{d}x.$$

However these inequalities need not hold if $g(x)$ is not positive on the whole interval. For example, if

$$a = -1, \qquad b = 1, \qquad f(x) = x \qquad g(x) = x$$

then

$$\int_a^b g(x)\,\mathrm{d}x = \int_{-1}^1 x\,\mathrm{d}x = 0, \qquad \int_a^b f(x)g(x)\,\mathrm{d}x = \int_{-1}^1 x^2\,\mathrm{d}x = \frac{2}{3}$$

and

$$0 \leqslant \frac{2}{3} \leqslant 0$$

is untrue, whatever our choices for m and M.