Solution (#1278) Let $f(x)$ be a decreasing positive function defined for $x \geqslant 0$. As seen in the graph below, we have for each $k = 1, 2, \ldots, n$ that

$$f(k) \geqslant \int_{k-1}^{k} f(x)\,\mathrm{d}x \geqslant f(k-1).$$

Hence, summing these N inequalities, we find

$$\sum_{k=0}^{n-1} f(k) \geqslant \sum_{k=1}^{n} \int_{k-1}^{k} f(x)\,\mathrm{d}x = \int_{0}^{n} f(x)\,\mathrm{d}x \geqslant \sum_{k=1}^{n} f(k).$$

In the diagram below we see solid rectangles have area $\sum_{k=0}^{3} f(k)$ and the dashed rectangles have area $\sum_{k=1}^{4} f(k)$.