

Solution (#1319) We have

$$4 \cos x + 3 \sin x = 5 \cos(x - \alpha)$$

where $\alpha = \tan^{-1}(3/4)$. Recall that

$$\int \sec x \, dx = \ln |\sec x + \tan x| + \text{const.}$$

So

$$\int \frac{dx}{4 \cos x + 3 \sin x} = \frac{1}{5} \ln \left| \frac{5 + 4 \sin x - 3 \cos x}{4 \cos x + 3 \sin x} \right| + \text{const.}$$