Solution (#1321) We set

$$I_1 = \int \frac{\sin x \ \mathrm{d}x}{\sin x + \cos x}; \qquad I_2 = \int \frac{\cos x \ \mathrm{d}x}{\sin x + \cos x},$$

and note

$$\begin{aligned} I_1 + I_2 &= \int \left(\frac{\sin x + \cos x}{\sin x + \cos x} \right) \ \mathrm{d}x = \int \mathrm{d}x = x + \text{const.}; \\ I_2 - I_1 &= \int \left(\frac{\cos x - \sin x}{\sin x + \cos x} \right) \ \mathrm{d}x = \ln |\sin x + \cos x| + \text{const.} \end{aligned}$$

as the denominator differentiates to the numerator. Solving these two simultaneous equations in I_1 and I_2 we have

$$\begin{aligned} I_1 &= -\frac{1}{2} \ln |\sin x + \cos x| + \frac{x}{2} + \text{const.}; \\ I_2 &= \frac{1}{2} \ln |\sin x + \cos x| + \frac{x}{2} + \text{const.}. \end{aligned}$$

Now set

$$J_1 = \int \frac{\sin x \ \mathrm{d}x}{a \sin x + b \cos x}; \qquad J_2 = \int \frac{\cos x \ \mathrm{d}x}{a \sin x + b \cos x}.$$

Then

$$aJ_1 + bJ_2 = \int \mathrm{d}x = x + \text{const.}$$

and

$$aJ_2 - bJ_1 = \int \left(\frac{a \cos x - b \sin x}{a \sin x + b \cos x} \right) \ \mathrm{d}x = \ln |a \sin x + b \cos x| + \text{const.}$$

Again solving these two simultaneous equations in J_1 and J_2 we find

$$\begin{aligned} J_1 &= \frac{-b}{a^2 + b^2} \ln |a \sin x + b \cos x| + \frac{ax}{a^2 + b^2} + \text{const.}; \\ J_2 &= \frac{a}{a^2 + b^2} \ln |a \sin x + b \cos x| + \frac{bx}{a^2 + b^2} + \text{const.} \end{aligned}$$