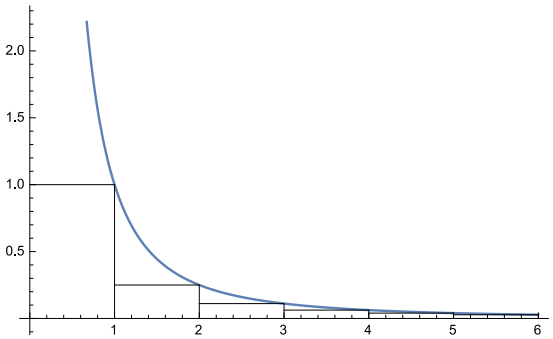
Solution (#1327) Below is a graph of $y = 1/x^2$ for $x > 0$. Note that the rectangles sit below the graph and so have a smaller area.



The rectangles each have unit base and heights of 1, $\frac{1}{4}$, $\frac{1}{9}$, ... The area under the graph for $x \geqslant 1$ equals

$$\int_1^\infty \frac{\mathrm{d}x}{x^2} = \left[\frac{-1}{x}\right]_1^\infty = 1$$

and so is finite. Hence

$$\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots \leqslant 1$$

and

$$S = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots \leqslant 1 + 1 = 2.$$