

Solution (#1350) By IBP

$$\begin{aligned}\int x^n (\ln x)^m \, dx &= \frac{x^{n+1}}{n+1} (\ln x)^m - \int \frac{x^{n+1}}{n+1} m x^{-1} (\ln x)^{m-1} \, dx \\ &= \frac{x^{n+1} (\ln x)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln x)^{m-1} \, dx.\end{aligned}$$

Hence

$$\begin{aligned}\int x^3 (\ln x)^2 \, dx &= \frac{x^4 (\ln x)^2}{4} - \frac{2}{4} \int x^3 \ln x \, dx \\ &= \frac{x^4 (\ln x)^2}{4} - \frac{1}{2} \left\{ \frac{x^4 \ln x}{4} - \frac{1}{4} \int x^3 \, dx \right\} \\ &= \frac{x^4 (\ln x)^2}{4} - \frac{x^4 \ln x}{8} + \frac{x^4}{32} + \text{const..}\end{aligned}$$